866. Proposed by Edwin F. Sampang, Manilla, Philippines..

Find $\frac{z}{y+z}$, given that $\frac{z}{x+y} = a$ and $\frac{y}{x+z} = b$.

Solution by Arkady Alt , San Jose ,California, USA.

Solution1.

Denoting $t := \frac{z}{y}$ and $s := \frac{x}{y}$ we obtain

$a = \frac{ty}{sy+y} = \frac{t}{s+1}$ and $b = \frac{y}{sy+ty} = \frac{1}{s+t}$. Hence, $s = \frac{t}{a} - 1 = \frac{1}{b} - t \Rightarrow$

$t\left(1+\frac{1}{a}\right) = 1 + \frac{1}{b} \Leftrightarrow t = \frac{a(b+1)}{b(a+1)}$ and, therefore,

$$\frac{z}{y+z} = \frac{t}{t+1} = \frac{a(b+1)}{a(b+1)+b(a+1)} =$$

$$\frac{a(b+1)}{2ab+a+b}.$$

Solution 2.

Let $u := \frac{x}{x+y+z}, v := \frac{y}{x+y+z}$ and $w := \frac{z}{x+y+z}$ then $u+v+w = 1$ and

$a = \frac{z}{x+y} = \frac{w}{u+v} = \frac{w}{1-w}, b = \frac{y}{x+z} = \frac{v}{u+w} = \frac{v}{1-v}$.

Hence, $\frac{1}{a} = \frac{1}{w} - 1 \Leftrightarrow w = \frac{a}{1+a}$, $\frac{1}{b} = \frac{1}{v} - 1 \Leftrightarrow v = \frac{b}{1+b}$ and

$$\frac{z}{y+z} = \frac{w}{v+w} = \frac{\frac{a}{1+a}}{\frac{b}{1+b}+\frac{a}{1+a}} = \frac{a(1+b)}{b(1+a)+a(1+b)} = \frac{a(b+1)}{2ab+a+b}.$$